FE simulation of metal foams based on the macroscopic approach of the Theory of Porous Media

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Abstract

Based on the increasing importance of metal foams in industrial applications, numerical computations are needed for an effective simulation of the material behaviour in general boundary value problems, for example energy absorbtion or elasto-plastic deformation problems.
Proceeding from either real or virtual averaging processes, the macroscopic description of gas-saturated or empty metal foams results in the continuum mechanical framework of the Theory of Porous Media (TPM). The porous solid matrix is described, in the constitutive range, by an elasto-plasticity law proceeding from a single-surface yield function to bound the elastic domain.

1. Theory of Porous Media

The material behaviour of immiscible constituents can be described by the Theory of Porous Media (TPM) [1]. This theory is defined as the theory of mixtures extended by the concept of volume fractions. Averaging the real structure toward a homogenised model, individual microscopic reactions depending on different pore sizes and shapes as well as on different cell wall reactions could be treated in a macroscopic way.
The kinematics of porous media depend on the assumption that particles X^α of the constituent φ^α (α = S: solid skeleton, α = F: pore-gas) of the medium under consideration occupy the same volume element $\mathrm{d}v$ which is defined as the sum of the partial volume elements $\mathrm{d}v^\alpha$ of all constituents (superimposed continua).
Material incompressibility of any constituent means incompressibility in the materials micro ranges and does not imply macroscopic incompressibility, i.e. even if the real density $\rho^{\alpha R}$ (material density) of a constituent φ^α is constant, the partial densities ρ^α can still change through changes in the volume fractions n^α. These relations are given by the definition of the volume fractions, the saturation condition and the partial density formulation:

$$n^\alpha = \frac{\mathrm{d}v^\alpha}{\mathrm{d}v}, \qquad \sum_\alpha n^\alpha = 1, \qquad \rho^\alpha = n^\alpha \rho^{\alpha R}.$$

Based on the principles of mixture theories, each constituent follows its own motion χ_α, where each particle X^α is assigned to its own reference position $\mathbf{X}_\alpha$ at time $t = t_0$.

J. Banhart, M.F. Ashby, N.A. Fleck: *Metal Foams and Porous Metal Structures.* © MIT Verlag (1999)

The deformation gradients and their inverses are represented via

$$\mathbf{F}_\alpha = \frac{\partial \mathbf{x}}{\partial \mathbf{X}_\alpha} = \mathrm{Grad}_\alpha \mathbf{x}, \qquad \mathbf{F}_\alpha^{-1} = \frac{\partial \mathbf{X}_\alpha}{\partial \mathbf{x}} = \mathrm{grad}\, \mathbf{X}_\alpha,$$

where $\mathrm{Grad}_\alpha(..)$ means partial differentiation with respect to the reference position $\mathbf{X}_\alpha$ and $\mathrm{grad}\,(..)_\alpha$ defines the partial differentiation related to the actual position $\mathbf{x}$.
By the use of the TPM for an immiscible mixture of constituents φ^α, each constituent can be described by an individual balance equation taking into account the necessary interactions between all constituents. The mixture equations are then described as the sum of the balance equations of all constituents and a constraint condition related to the interactions.
Because the body does not know, whether it is a mixture or not, the mixture balance equation must have the same form as the respective balance equations of a single body. So the properties of the mixture follow from the properties of the constituents without any additional assumption:

balance equation	local form	mixture constraint
mass:	$(\rho^\alpha)'_\alpha + \rho^\alpha \mathrm{div}\, \overset{\prime}{\mathbf{x}}_\alpha = \hat{\rho}^\alpha$	$\sum_{\alpha=S,F} \hat{\rho}^\alpha = 0$
momentum:	$\rho^\alpha \overset{\prime\prime}{\mathbf{x}}_\alpha = \mathrm{div}\, \mathbf{T}^\alpha + \rho^\alpha \mathbf{b}^\alpha + \hat{\mathbf{p}}^\alpha$	$\sum_{\alpha=S,F} (\hat{\mathbf{p}}^\alpha + \hat{\rho}^\alpha \overset{\prime}{\mathbf{x}}_\alpha) = 0$

In these equations $(..)'_\alpha$ denotes the material time derivative following the motion of the constituent φ_α, $\hat{\rho}^\alpha$ is the mass exchange (here: no mass exchanges are assumed: $\hat{\rho}^\alpha = 0$), $\mathbf{T}^\alpha$ is the partial *Cauchy* stress tensor, $\mathbf{b}^\alpha$ is the body force density (here: $\mathbf{b}^S = \mathbf{b}^F = \mathbf{b}$) and $\hat{\mathbf{p}}^\alpha$ represents the momentum production term of φ^α.
The response functions must be compatible with the second law of thermodynamics. Therefore the constitutive equations must reflect the restrictions imposed on the model by the dissipation principle. Only the main results from the procedure of deriving thermodynamical restrictions are given here. The reader who is interested in further details is referred to [1].
For the incompressible solid skeleton fully saturated by a compressible pore-fluid, the partial stress tensors as well as the interaction relation can be divided into a pressure depending and an extra part:

$$\mathbf{T}^\alpha = -n^\alpha p \mathbf{I} + \mathbf{T}^\alpha_E, \qquad \hat{\mathbf{p}}^F = -\hat{\mathbf{p}}^S = p\, \mathrm{grad}\, n^F + \hat{\mathbf{p}}^F_E,$$

where $\mathbf{I}$ is the second order identity tensor weighted by the effective fluid pressure p and the volume fraction n^α of the related phase φ^α, while $\mathbf{T}^\alpha_E$ represents the extra stresses and $\hat{\mathbf{p}}^F_E$ the extra part of the interaction force. These extra terms has to be specified by additional constitutive relations. Thus, it is assumed that the gas phase reacts like an ideal gas, the extra part of the interaction force is related to *Darcy*'s law when being inserted in the fluid momentum balance, the fluid extra stresses are assumed to be negligible and only the considerations for the solid extra stresses has to be defined in relation to the free energy function ψ^S of the solid phase. This can be summarised by

$$p = \rho^{FR} \bar{R}^F \theta, \qquad \hat{\mathbf{p}}^F_E = -\frac{(n^F)^2 \gamma^{FR}}{k^F} \mathbf{w}^F, \qquad \mathbf{T}^F_E \approx \mathbf{0}, \qquad \mathbf{T}^S_E = \rho^S \frac{\partial \psi^S}{\partial \mathbf{F}_S} \mathbf{F}^T_S.$$

In the above equations $\bar{R}^F$ is the specific gas constant, θ the absolute temperature, k^F is the *Darcy* permeability coefficient and γ^{FR} is the effective fluid weight.

2. Elasto-Plasticity

Proceeding from the assumptions of an elasto-plastic solid material and geometrically non-linear deformations, the general response of the solid skeleton stress is affected by the multiplicative decomposition of the deformation gradient $\mathbf{F}_S = \mathbf{F}_{Se}\mathbf{F}_{Sp}$ into an elastic and a plastic part. This implies the existence of only two proper spatial velocity gradients, namely $\mathbf{L}_S$ as the spatial velocity gradient and $\hat{\mathbf{L}}_{Sp}$ as the plastic velocity gradient, see [1]. Their symmetric parts $\mathbf{D}_S$ characterises the solid deformation rate of the spatial configuration and $\hat{\mathbf{D}}_{Sp}$ the purely plastic deformation rate of the intermediate configuration (terms corresponding to the intermediate configuration are denoted by $(\hat{\cdot})$).
By the use of the concept of *Lie* derivatives (or *Oldroyd* derivatives, respectively) and some further transformation conditions, an additively decomposition of the total strains results. Thus, a modified *Neo-Hooke*-model defined by [2] and having been extended for finite elasto-plasticity by [3] is given to specify the solid skeleton extra stress, via:

$$\boldsymbol{\tau}_E^S = \mu^S(\mathbf{B}_{Se} - \mathbf{I}) + \lambda^S(1 - n_p^S)^2 \left(\frac{J_{Se}}{1 - n_p^S} - \frac{J_{Se}}{J_{Se} - n_p^S} \right) \mathbf{I}.$$

Therein, $\boldsymbol{\tau}_E^S$ represents the extra *Kirchhoff* stress, μ^S and λ^S are the *Lamé* constants with respect to the solid skeleton, $\mathbf{B}_S$ is the elastic part of the left *Cauchy-Green* strain tensors, J_{Se} is the *Jacobian* with respect to the elastic deformation, and n_p^S is the plastic solid volume fraction with respect to the intermediate configuration. This formulation ensures that the point of compaction is correctly represented.
The plasticity is described with respect to the intermediate configuration. The different postulates and results of the plastic response can be transformed by use of the respective push-forward and pull-back transformations from the intermediate configuration towards the actual configuration or the reference configuration, if required.
As it is known from theories of constrained materials, only the extra stresses are allowed to enter the plastic response. Following this, a temperature independent general yield function for porous matrices can be introduced by a single surface yield condition [1],

$$F(\hat{\mathrm{I}}, \hat{\mathrm{II}}_D, \hat{\mathrm{III}}_D, \boldsymbol{q}, \boldsymbol{r}) = \sqrt{\Psi_1 \hat{\mathrm{II}}_D \left(1 + \gamma \frac{\hat{\mathrm{III}}_D}{\hat{\mathrm{II}}_D^{3/2}} \right)^m + \frac{1}{2}\alpha \hat{\mathrm{I}}^2 + \delta^2 \hat{\mathrm{I}}^4 + \Psi_2 \beta \hat{\mathrm{I}} + \varepsilon \hat{\mathrm{I}}^2} - \kappa = 0,$$

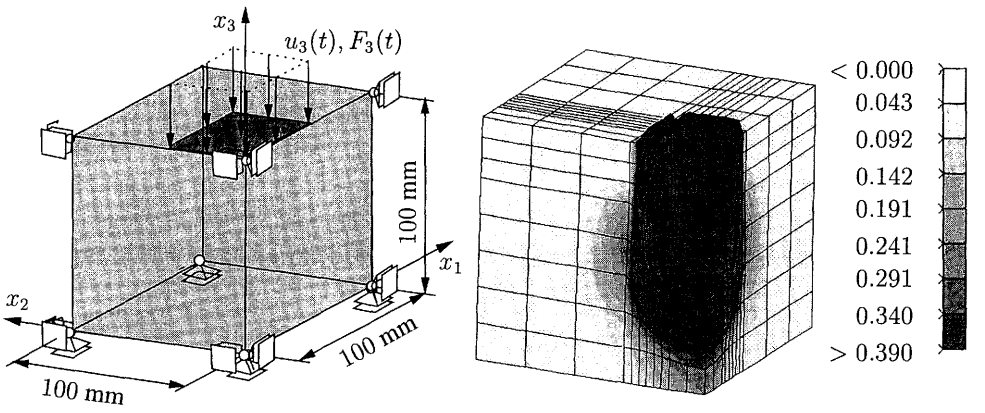
where $\hat{\mathrm{I}}$ is the first invariant, $\hat{\mathrm{II}}_D$ is the negative second deviatoric invariant and $\hat{\mathrm{III}}_D$ is the third deviatoric invariant of $\hat{\boldsymbol{\tau}}_E^S$, $\boldsymbol{q} = (\alpha, \beta, \gamma, \delta, \varepsilon, \kappa, m)$ is the parameter vector representing the dependence on the deformation history in the isotropic hardening case, and $\boldsymbol{r} = (\Psi_1, \Psi_2)$ includes the parameters representing the dependence on the actual deformation describing the structural hardening.
The set of material parameters must be fitted to experimental results from triaxial and biaxial tests. As far as porous materials are concerned, a general non-associated flow rule must be introduced to govern the plastic rate of the deformation tensor $\hat{\mathbf{D}}_{Sp}$ depending on the plastic potential G and the proportional factor Λ:

$$\hat{\mathbf{D}}_{Sp} = \Lambda \frac{\partial G}{\partial \hat{\boldsymbol{\tau}}_E^S}, \qquad G(\hat{\mathrm{I}}, \hat{\mathrm{II}}_D, \mathbf{q}, \mathbf{r}) = \sqrt{\Psi_1 \hat{\mathrm{II}}_D + \frac{1}{2}\alpha \hat{\mathrm{I}}^2 + \delta^2 \hat{\mathrm{I}}^4 + \Psi_2 \beta \hat{\mathrm{I}} + \varepsilon \hat{\mathrm{I}}^2} = 0.$$

For the numerical computations, the equation system resulting from the weak forms of the balance equations as well as the local plasticity equation system has to be solved, see [4].

3. Example: Indentation test

As an example of the usage of metal foams in real applications, a simple indentation test is presented. Herein, a part of an aluminium foam block is loaded by a rigid stamp, see sketch below.



After an indentation way of 50 percent of the specimen's height, the norm of the plastic strains are shown. At this state of deformation, some elements have reached the point of compaction and thus react like elements of single phase material in further loading. The high effective stresses are localised in the loaded region, especially at the edges of the indented area. Furthermore, the plastification of the foam material is also very local in this indented area which also can be observed by experiments.
Further work is the specification of material parameters of different foam materials and the verification of numerical simulations with the corresponding experiments.

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